

BA-94-46
UMD-PP-94-99
hep-ph/9409381

A Hint From the Inter-Family Mass Hierarchy: Two Vector-Like Families in the TeV Range

K.S. Babu¹

*Bartol Research Institute
University of Delaware
Newark, DE 19716*

Jogesh C. Pati²

*Department of Physics
University of Maryland
College Park, MD 20742*

Hanns Stremnitzer

*Institute for Theoretical Physics
University of Vienna
Boltzmannngasse 5, A-1090
Vienna, Austria*

¹Work supported in part by a grant from the DOE

²Work supported in part by a grant from the NSF

Abstract

Two vector-like families $Q_{L,R} = (U, D, N, E)_{L,R}$ and $Q'_{L,R} = (U', D', N', E')_{L,R}$ with masses of order 1 TeV, one of which is a doublet of $SU(2)_L$ and the other a doublet of $SU(2)_R$, have been predicted to exist, together with the three observed chiral families, in the context of a viable and economical SUSY preon model. The model itself possesses many attractive features which include explanations of the origins of (i) diverse mass-scales, (ii) family-replication, (iii) protection of the masses of the composite quarks and leptons compared to their compositeness scale and (iv) inter-family mass-hierarchy. The existence of the two vector-like families – a prediction of the model – turns out to be crucial especially for an explanation of the inter-family mass-hierarchy (IFMH). Given the simplicity of the explanation, the observed IFMH in turn appears to us to be a strong hint in favor of the existence of the two vector-like families.

This paper is devoted to a detailed study of the expected masses, mixings and decay modes of the fermions belonging to the two vector-like families, in the context of the SUSY preon model, with the inclusion of the renormalization-effects due to the standard model gauge interactions. Including QCD renormalization-effects, the masses of the vector-like quarks are expected to lie in the range of 500 GeV to about 2.5 TeV, while those of the vector-like leptons are expected to be in the range of 200 GeV to 1 TeV. Their mass pattern and decay modes exhibit certain distinguishing features and characteristic signals. For example, when the LHC and, possibly a future version of the SSC are built, pair-production of the vector-like quarks would lead to systems such as $(b\bar{b} + 4Z + W^+W^-)$ and $(b\bar{b} + 2Z + W^+W^-)$, while an e^-e^+ linear collider (NLC) of suitable energy can produce appreciably a single neutral heavy lepton N together with ν_τ , followed by the decay of N into $(Z + \nu_\tau) \rightarrow (e^-e^+) + \nu_\tau$. This last signal may conceivably materialize even at LEP 200 if N is lighter than about 190 GeV

I. Introduction

Searches for the Higgs bosons and supersymmetry – related directly and/or indirectly to electroweak symmetry breaking – are among the commonly cited motivations, deservedly so, for the building of the hadronic colliders like the LHC, a future version of the now extinct SSC, as well as the next e^-e^+ linear collider NLC. The discovery of the Higgs boson(s) will clearly shed light on the problem of electroweak symmetry breaking and thereby on the origin of the masses of W and Z , while that of supersymmetry will provide assurance on the common understanding of the gauge–hierarchy problem. But by themselves, none of these discoveries would shed light on an understanding of the inter–family mass–hierarchy (IFMH) and therefore on the origin of the masses of the quarks and the leptons, of which all matter is made. As regards this last issue, while there are a few explanations, we believe that there is a particularly simple one which deserves attention. For this simple explanation of the IFMH to hold, there must exist two “vector–like” quark–lepton families $Q_{L,R}$ and $Q'_{L,R}$ with masses of order 1 TeV, where $Q_{L,R}$ couple vectorially –i.e., in a parity conserving manner–to W_L ’s, while $Q'_{L,R}$ couple vectorially (assuming a left–right symmetric gauge theory) to W_R ’s.

As it turns out, two such vector–like families and the associated fermion mass matrix, providing an explanation of the IFMH, arise in a compelling manner in a supersymmetric preon model [1-3]. Since a search for these two vector–like families need facilities like the SSC and the LHC, by way of emphasizing the dire need for the building of such accelerators, we first recall the *essential role* which these two families play in providing an explanation of the inter–family mass hierarchy. The main purpose of the paper is to spell out the expected properties of these two families – i.e., their masses, mixings

and decay modes – in some detail. These should facilitate their search, if and when the LHC, a possible new version of the SSC and/or the NLC are built.

Before proceeding further, it is useful to recall one crucial distinction between the chiral and the two vector-like families. Since Q_L and Q_R couple symmetrically to $SU(2)_L \times U(1)_Y$ gauge bosons, the mass term $(\overline{Q}_L Q_R + h.c)$ and likewise $(\overline{Q}'_L Q'_R + h.c)$ preserve $SU(2)_L \times U(1)_Y$. In a class of models, the masses of these vector-like families, protected by additional symmetries (see below), turn out naturally to be of order 1 TeV, rather than being of order Planck mass or some other superheavy scale. Because their masses are $SU(2) \times U(1)$ -symmetric, however, the oblique parameters S, T and U [4], or equivalently ρ (or ϵ_1) and ϵ_3 [5] do not receive contributions from these vector-like families, in the leading approximation. As a result, the prevailing set of measurements of the electroweak parameters, despite their precision, are not sensitive enough to the existence of vector-like families [6]. This is unlike the case of a fourth chiral family which is slightly disfavored and a technicolor family which seems to be excluded (at least in its simple form) by the measurement of the S, T and U -parameters. This leads one to infer that if new families beyond the three are yet to be found, they are more likely to exhibit vectorial rather than chiral couplings to W_L 's and W_R 's (at least in their canonical forms before $Q - Q'$ mixings) [6].

The reason why we take the possible existence of two vector-like families $Q_{L,R} = (U, D, N, E)_{L,R}$ and $Q'_{L,R} = (U', D', N', E')_{L,R}$ with masses of order 1 TeV seriously is two-fold. First of all, as mentioned above, they arise as a *compelling prediction* of a SUSY composite model [1], which seems to possess many attractive features. These include an understanding of (a) the origin of family replication [2], (b) protection of the masses of composite quarks and

leptons, compared to their compositeness scale [7], and (c) the origin of the diverse mass scales—from M_{Planck} to m_ν [1]. In addition, the model provides several testable predictions [1-3], by which it can be excluded, if it is wrong.

Second of all, *the existence of the two vector-like families is found to be crucial in the model to the very origin of the masses of the three chiral families and simultaneously, to an understanding of the inter-family mass-hierarchy.* Both features turn out to have their origin [1,3] in a spontaneously induced see-saw pattern for the 5×5 mass matrix of the three chiral and the two vector-like families, in which the direct mass terms of the three chiral families $m^{(0)}(q_L^i \rightarrow q_R^j)$ vanish naturally owing to underlying symmetries of the theory, barring small corrections that are less than or of order 1 MeV. Although the compositeness scale is determined on various grounds to be around 10^{11} GeV, owing to protection by a chiral symmetry (which is the non-anomalous R -symmetry), the two vector-like families turn out to acquire masses of order only 1 TeV. The chiral families acquire masses primarily only by their mixings with the two vector-like families. One *general consequence* of such a mass-matrix, which follows simply from its rank, (see details later), is that *one linear combination of the three chiral families, is guaranteed to remain massless, barring corrections of order 1 MeV*, which is thus identified with the electron family. At the same time, the heaviest chiral fermion (top) acquires a mass of nearly 100-170 GeV and the masses of the fermions belonging to the muon family lie intermediate in the range of 100-1500 MeV [3]. In this way, the see-saw mass pattern of the type generated in the SUSY composite model provides a simple resolution of the puzzle of inter-family mass-hierarchy. In particular, it explains the large hierarchy between m_e and m_t . Since such a pattern would not be possible without the two vector-like families, the observed inter-family hierarchy seems to be a

strong hint in favor of the existence of two such families with masses of order 1 TeV. (In the sequel, it will be clear as to why their number will have to be precisely two—no less and no more).

Due to the mixing of the chiral with the two vector-like families the model suggests [3,1] a sizeable strength of $\Delta m(D - \bar{D})$ and observable rates for $\mu \rightarrow 3e$ and especially for $t \rightarrow Zc$. Such mixings also lead to a lengthening of the tau lifetime and simultaneously to a correlated small decrease in the LEP neutrino counting N_ν from 3 [6]. Barring these small indirect effects, these new families can, of course, be discovered only provided machines like the SSC and the LHC and possibly TeV-range e^+e^- colliders are built.

With this in view, we spell out certain characteristics of the two vector-like families—i.e., their expected masses, mixings and decay modes in the context of Ref. 1-3. These turn out to possess some rather unexpected and interesting features because of the see-saw pattern of the fermion mass-matrix on the one hand and the differing renormalization group effects on the entries in the mass matrix involving $Q_{L,R}$ as opposed to those involving $Q'_{L,R}$, on the other hand. The differences in the renormalization effects arise because $Q_{L,R}$ couple to W_L and $Q'_{L,R}$ do not. Among the characteristic signals, we find that the production and subsequent decays of certain pairs of the heavy quarks would give rise to systems such as $b\bar{b} + 4Z + 2W$ and $b\bar{b} + 2Z + 2W$. With the Z -boson decaying via charged lepton pairs, these decay modes would provide distinctive signals and would thereby facilitate the search for these new vector-like families, at future hadronic colliders.

An additional intriguing signal is the single production of a neutral heavy lepton (N) together with ν_τ in an e^+e^- machine of appropriate energy, followed by the decay of N into $(\nu_\tau + Z) \rightarrow (\nu_\tau + e^+e^-)$. This last signal may even be visible at LEP 200 if N is lighter than about 190 GeV.

II. Masses and Mixings of the Vector–Like Families

The masses of the five–family system – three chiral ($q_{L,R}^i$) and two vector–like ($Q_{L,R}$ and $Q'_{L,R}$) – arise as follows. (For details we refer the reader to Ref. 1 and 3.)

It is assumed that as the asymptotically free metacolor force becomes strong at a scale $\Lambda_M \sim 10^{11} \text{ GeV}$ (this corresponds to an input effective coupling $\overline{\alpha}_M \simeq 1/27$ near the Planck scale), it forms a few SUSY–preserving and also SUSY–breaking condensates. Among the latter are the fermionic condensates consisting of the metagaugino pair $\langle \vec{\lambda} \cdot \vec{\lambda} \rangle$ and the preonic fermion pairs $\langle \overline{\psi}^f \psi^f \rangle_{f=u,d}$ and $\langle \overline{\psi}^c \psi^c \rangle_{c=(r,y,b),l}$, where f and c denote the flavor and the color attributes of the preons [1]. Noting that in the model under consideration, a dynamical breaking of SUSY would be forbidden in the absence of gravity, owing to the Witten index theorem [8], it has been argued that each of these fermion condensates which happen to break SUSY, must be damped by (Λ_M/M_{Pl}) [7], so that they would vanish in the absence of gravity (i.e., in the limit of $M_{Pl} \rightarrow \infty$). Thus, one has:

$$\begin{aligned} \langle \overline{\psi}^a \psi^a \rangle &= a_{\psi_a} \Lambda_M^3 (\Lambda_M/M_{Pl}) \\ \langle \vec{\lambda} \cdot \vec{\lambda} \rangle &= a_{\lambda} \Lambda_M^3 (\Lambda_M/M_{Pl}) \end{aligned} \quad (1)$$

$a = u, d, r, y, b, l$. The coefficients a_{λ} and a_{ψ_a} are apriori expected to be of order unity, although a_{λ} is expected to be bigger than a_{ψ_a} by a factor of 3 to 10 (say), because λ ’s are in the adjoint and ψ ’s in the fundamental representation of the metacolor group. One can argue that even a bosonic condensate $\langle \phi^* \phi \rangle$, if it forms, would be damped by powers of (Λ_M/M_{Pl}) [7].

Given these fermionic condensates, the vector-like families Q and Q' acquire relatively heavy Dirac masses $O[a_\lambda \Lambda_M (\Lambda_M/M_{Pl})] \sim O(1 \text{ TeV})$ through the metagaugino condensate $\langle \vec{\lambda} \cdot \vec{\lambda} \rangle$, which does not distinguish between up and down flavors and between quarks and leptons. Owing to extra symmetry associated with the composite chiral families $(q_{L,R}^i)$, however, their direct mass-terms $m_{\text{dir}}^{(0)}(q_L^i \rightarrow q_R^j)$ cannot be induced through either $\langle \vec{\lambda} \cdot \vec{\lambda} \rangle$ or $\langle \bar{\psi} \psi \rangle$. These receive small contributions through e.g., products of chiral symmetry breaking $\langle \bar{\psi} \psi \rangle$ and bosonic $\langle \phi^* \phi \rangle$ condensates, which are thus damped by $(\Lambda_M/M_{Pl})^2$ and are $\leq O(1 \text{ MeV})$.

The chiral families (especially the μ and the τ -families), acquire their masses almost entirely through their off-diagonal *mixings* with the vector-like families, which are induced by the $\langle \bar{\psi} \psi \rangle$ -condensates. As a result, the mass matrices for the five-family system (barring electroweak corrections and $m_{\text{dir}}^{(0)} \leq 1 \text{ MeV}$) have the following form at Λ_M [1-3]:

$$M_{f,c}^{(o)} = \frac{\overline{q_R^i}}{\overline{Q_R}} \begin{pmatrix} O & X\kappa_f & Y\kappa_c \\ Y'^\dagger \kappa_c & \kappa_\lambda & O \\ X'^\dagger \kappa_f & O & \kappa_\lambda \end{pmatrix} \quad . \quad (2)$$

Here $f = u$ or d and $c = (r, y, b)$ or l , thus this form of the mass matrix applies to four sectors q_u, q_d, l and ν . The superscript $i = 1, 2, 3$ runs over three chiral families. The entries X, Y, X' and Y' are column matrices in the space of these three chiral families with entries of order unity. In the above $\kappa_\lambda \equiv O(a_\lambda) \Lambda_M (\Lambda_M/M_{Pl}) \sim O(1 \text{ TeV})$; $\kappa_{f,c} \equiv O(a_{\psi_{f,c}}) \Lambda_M (\Lambda_M/M_{Pl}) = [O(a_{\psi_{f,c}})/O(a_\lambda)] \kappa_\lambda$. Following remarks made above, we expect

$\kappa_{f,c} = O(1/3 \text{ to } 1/10) \kappa_\lambda$. Thus the Dirac mass matrices of all four sectors have a natural see-saw structure, and the two vector-like families acquire

masses $\simeq \kappa_\lambda \sim O(1 \text{ TeV})$, while the three chiral families acquire lighter masses $\sim (X_i Y_i'^* + X_i'^* Y_i)(\kappa_f \kappa_c / \kappa_\lambda) \ll \kappa_\lambda$ at Λ_M [3].

In the absence of electroweak corrections, which are typically of order (1–10)%, we have $X = X'$ and $Y = Y'$. Furthermore, the same X, Y and κ_λ apply to all four sectors: q_u, q_d, l and ν . A still further reduction in effective parameters is achieved by rotating the chiral fields q_L^i and q_R^i so that without loss of generality the row matrices $Y^T = Y'^T$ can be brought to the simple form $(0, 0, 1)$ and $X^T = X'^T$ can be brought to the form $(0, p, 1)$ with redefined κ_f and κ_c . One can argue plausibly on the basis of preon–diagram that p is less than one, but not very much smaller than one–i.e, $p \approx 1/2$ to $1/10$ is reasonable.

Given these rotated forms of X and Y , *it is clear that one family is massless, barring corrections of order 1 MeV* which arise from the direct mass terms. This one is identified with the electron–family. The masses of the muon–family, evaluated at Λ_M are given by $(m_{\mu''})^{f,c} \simeq (p^2/2)(\kappa_f \kappa_c / \kappa_\lambda)$, while those of the tau family are given by $(m_{\tau''})^{f,c} \simeq 2(\kappa_f \kappa_c / \kappa_\lambda)$. The μ/τ mass ratio is thus given by $p^2/4$. For a value of $p \approx 1/3$ to $1/4$ (say), which is not too small and reasonable, one thus obtains a large hierarchy in the μ/τ mass-ratio of about $1/40$ to $1/64$, as observed. In this way, the 5×5 see-saw mass-matrix, with the approximate zeros dictated by the symmetry of the theory, provides a very simple explanation of the inter–family hierarchy: $m_{u,d,e} \ll m_{c,s,\mu} \ll m_{t,b,\tau}$ with $m_e \sim O(1 \text{ MeV})$ and $m_t \sim O(100 \text{ GeV})$. In particular, it explains why (m_e/m_t) is small ($\sim 10^{-5}$). The role of the two vector–like families is, of course, crucial to obtain such an explanation. *This is the reason why we expressed in the introduction that the two vector–like families may hold the key to an understanding of the inter–family hierarchy.*

In the present note, we are primarily concerned with the masses, mixings

and decay modes of the two heavy families. For this it is necessary to retain their mixing at least with the tau family. For the sake of simplicity, we will ignore their mixings with the lighter electron and the muon families. This is, of course, a very good approximation, especially for the electron family. Even for the muon family, the $\mu - Q$ mixing angle is smaller than the $\tau - Q$ mixing angle by the factor of $p \approx 1/3$ to $1/4$ [3].

Thus, ignoring the mixings with the e and the μ -families, which are defined only after the transformation of the X and Y -matrices to the simple forms as mentioned above, the truncated 3×3 mass matrix of the tau and the two vector-like families, evaluated at Λ_M , is given by

$$\hat{M}_{f,c}^{(o)} = \frac{\overline{q_R^\tau}}{\overline{Q_R}} \begin{pmatrix} O & \kappa_f & \kappa_c \\ \kappa_c & \kappa_\lambda & O \\ \kappa_f & O & \kappa_\lambda \end{pmatrix}. \quad (3)$$

Noting that we expect $(\kappa_{f,c}/\kappa_\lambda) = O(1/5 \text{ to } 1/10)$, if we block-diagonalize this matrix to remove the $q - Q$ and $q - Q'$ entries, the τ -family would acquire a mass $\approx 2\kappa_f\kappa_c/\kappa_\lambda$ (as mentioned before), while the 2×2 sector involving (Q, Q') would have a symmetrical mass-matrix with two equal diagonal elements $(\kappa_\lambda + (\kappa_f^2 + \kappa_c^2)/2\kappa_\lambda)$ and off-diagonal elements $(\kappa_f\kappa_c/\kappa_\lambda)$ (to leading order in $\kappa_{f,c}/\kappa_\lambda$). From this, it would appear that the two heavy eigenstates would be given essentially by $(Q \pm Q')/\sqrt{2}$ corresponding to maximal mixing (barring small admixtures of tau family in each case). This is, however, greatly distorted due to electroweak renormalizations which distinguish between $Q_{L,R}$ and $Q'_{L,R}$, to which we now turn.

We evaluate the running of the electroweak and QCD coupling constants

$g_{1,2,3}$ in the regime spanning from $\Lambda_M \approx 3 \times 10^{11} \text{ GeV}$ to 1.5 TeV, using renormalization-group equations. In this regime, 3 chiral and two vector-like families, the two Higgs-like multiplets and their superpartners contribute to the β -functions. We use the familiar expressions for the mass-corrections—i.e. [9],

$$[m(Q)/m(\Lambda_M)] = \Pi[g_i^2(Q)/g_i^2(\Lambda_M)]^{b_i^m/2b_i} \quad (4)$$

where b_i and b_i^m are the coefficients that appear in the β_i and γ_i functions respectively,

$$b_i = \frac{1}{16\pi^2} \left[-\frac{11}{3}t_2(V) + \frac{2}{3}t_2(F) + \frac{1}{3}t_2(S) \right] \quad (5)$$

$$b_i^m = \frac{3}{8\pi^2} C_2(R).k \quad (6)$$

with $C_2(R) = (N^2 - 1)/(2N)$ for $SU(N)$, while $C_2(R) = y^2$ for $U(1)$, where y is the normalized hypercharge. The factor k is 2/3 (1) for a SUSY (non-SUSY) gauge theory. The b_i 's are given by

$$b_3 = \frac{1}{16\pi^2}, \quad b_2 = \frac{5}{16\pi^2}, \quad b_1 = \frac{(53/5)}{16\pi^2} \quad (7)$$

$$b_3^m = \frac{1}{4\pi^2} \left(\frac{4}{3} \right), \quad b_2^m = \frac{1}{4\pi^2} \left(\frac{3}{4} \right) \quad . \quad (8)$$

b_1^m can take seven different values depending upon the hypercharges of the external pairs of fermions. Denoting these seven values of b_1^m by $\hat{a}_j \equiv (a_j/4\pi^2)$ ($j = 1 - 7$), the values of the a_j 's for the relevant pairs of fermions are listed below:

$$\begin{aligned} a_1(\bar{U}_R t_L, \bar{U}_R U_L, \bar{D}_R b_L, \bar{D}_R D_L) &= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{3}{5} = \frac{1}{60} \\ a_2(\bar{t}_R U'_L, \bar{U}'_R U'_L) &= \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{3}{5} = \frac{4}{15} \\ a_3(\bar{U}'_R t_L, \bar{b}_R D'_L, \bar{D}'_R D'_L) &= \left(\frac{-1}{3} \right), \left(\frac{-1}{3} \right) \cdot \frac{3}{5} = \frac{1}{15} \end{aligned}$$

$$\begin{aligned}
a_4(\bar{b}_R D_L, \bar{D}'_R b_L) &= \frac{1}{6} \cdot \left(\frac{-1}{3}\right) \frac{3}{5} = -\frac{1}{30} \\
a_5(\bar{E}_R \tau_L, \bar{N}_R \nu_{\tau L}, \bar{N}_R N_L, \bar{E}_R E_L) &= \left(\frac{-1}{2}\right) \cdot \left(\frac{-1}{2}\right) \cdot \frac{3}{5} = \frac{3}{20} \\
a_6(\bar{\tau}_R E_L, \bar{E}'_R \tau_L) &= \left(\frac{-1}{2}\right) \cdot (-1) \cdot \frac{3}{5} = \frac{3}{10} \\
a_7(\bar{E}'_R E'_L) &= (-1) \cdot (-1) \cdot \frac{3}{5} = \frac{3}{5}
\end{aligned} \tag{9}$$

The factor 3/5 is the usual normalization factor for the hypercharge y .

The mass renormalization group parameters $\eta_c, \eta_L, \eta_{1,2,3,4,5,6,7}$, (corresponding to $SU(3)^C$, $SU(2)_L$ and $U(1)_Y$) are defined as

$$\begin{aligned}
\eta_c &= \left[\frac{g_3^2(1.5 \text{ TeV})}{g_3^2(\Lambda_M)} \right]^{-b_3^m/2b_3} \\
\eta_L &= \left[\frac{g_2^2(1.5 \text{ TeV})}{g_2^2(\Lambda_M)} \right]^{-b_2^m/2b_2} \\
\eta_i &= \left[\frac{g_1^2(1.5 \text{ TeV})}{g_1^2(\Lambda_M)} \right]^{-\hat{a}_i/2b_1}, \quad i = 1 - 7.
\end{aligned} \tag{10}$$

To evaluate these factors, we choose as input values $g_1^2(M_Z) = 0.2136$, $g_2^2(M_Z) = 0.4211$, $g_3^2(M_Z) = 1.4828$, corresponding to the values of $\alpha = 1/127.9$, $\sin^2 \theta_W = 0.2333$ and $\alpha_3 = 0.118$ measured at M_Z at LEP [10]. With three chiral and two vector-like families along with two Higgs multiplets and all their superpartners contributing to the β and γ functions in the energy regime 1.5 TeV to Λ_M , the gauge couplings at $\Lambda_M = 3 \times 10^{11} \text{ GeV}$ are evaluated to be $g_1^2(\Lambda_M) = 0.5163$, $g_2^2(\Lambda_M) = 0.8192$, $g_3^2(\Lambda_M) = 1.5958$ [11]. Using (7)-(10), the resulting η factors are thus given by

$$\begin{aligned}
\eta_c &= 2.39, \quad \eta_L = 1.23, \quad \eta_1 = 1.003, \quad \eta_2 = 1.043 \\
\eta_3 &= 1.011, \quad \eta_4 = 0.995, \quad \eta_5 = 1.024, \quad \eta_6 = 1.049, \quad \eta_7 = 1.1.
\end{aligned} \tag{11}$$

As expected, QCD renormalization effect denoted by η_c is large. That due to $SU(2)_L$, denoted by η_L , is significant. Note, however, that the renormaliza-

tion factors due to the hypercharge interaction given by $\eta_1 - \eta_7$ are typically rather small, differing from unity by 1 to 10%.

In addition to the renormalization group running from Λ_M to the weak scale, the mass parameters will also receive QCD and electroweak radiative corrections at Λ_M , which lead to differences between X and X' and between Y and Y' (see eq. (2)). These corrections are typically of the order of 5-10%, which are to be compared with the renormalization effects due to running given in eq. (11). Note that the $SU(2)_L$ running factor η_L is much bigger than these preonic corrections at Λ_M , while the hypercharge running corrections are of similar magnitude. In what follows, we shall neglect the corrections arising from the preon diagrams, which should represent a good approximation for the masses and mixings involving the top family and the vector-like families. (These corrections at Λ_M do play an important role for the masses and mixings involving the e and the μ families, see Ref. 3.)

Inserting the renormalization factors into (3), the 3×3 sector of the up, down and charged lepton mass matrices involving the tau and the two vector-like families take the form

$$M_{up} = \frac{\bar{t}_R}{\bar{U}_R} \begin{pmatrix} 0 & \kappa_u \eta_3 & \kappa_r \eta_2 \\ \kappa_r \eta_L \eta_1 & \kappa_\lambda \eta_L \eta_1 & 0 \\ \kappa_u \eta_3 & 0 & \kappa_\lambda \eta_2 \end{pmatrix} \eta_c \quad (12)$$

$$M_{down} = \frac{\bar{b}_R}{\bar{D}_R} \begin{pmatrix} 0 & \kappa_d \eta_4 & \kappa_r \eta_3 \\ \kappa_r \eta_L \eta_1 & \kappa_\lambda \eta_L \eta_1 & 0 \\ \kappa_d \eta_4 & 0 & \kappa_\lambda \eta_3 \end{pmatrix} \eta_c \quad (13)$$

$$M_{lepton}^+ = \frac{\overline{E}_R}{\overline{E}'_R} \begin{pmatrix} 0 & \kappa_d \eta_6 & \kappa_l \eta_7 \\ \kappa_l \eta_L \eta_5 & \kappa_\lambda \eta_L \eta_5 & 0 \\ \kappa_d \eta_6 & 0 & \kappa_\lambda \eta_7 \end{pmatrix}. \quad (14)$$

In the neutral lepton sector, since ν_{τ_R} becomes superheavy ($m_{\nu_{\tau_R}} \sim \Lambda_M \sim 10^{11} \text{ GeV}$) [1,3,12], the corresponding mass matrix is a 2×3 matrix given by

$$M_{lepton}^0 = \frac{\overline{N}_R}{\overline{N}'_R} \begin{pmatrix} \kappa_l \eta_L \eta_5 & \kappa_\lambda \eta_L \eta_5 & 0 \\ \kappa_u & 0 & \kappa_\lambda \end{pmatrix}. \quad (15)$$

Since $\kappa_{f,c} \ll \kappa_\lambda$, the mass matrices of eq. (12-15) can be diagonalized by using the see-saw formula. The light mass eigenvalues corresponding to the chiral quarks and leptons of the tau family are given by

$$\begin{aligned} m_t &\simeq 2 \frac{\kappa_u \kappa_r}{\kappa_\lambda} \eta_c \eta_3, & m_b &\simeq 2 \frac{\kappa_d \kappa_r}{\kappa_\lambda} \eta_c \eta_4 \\ m_\tau &\simeq 2 \frac{\kappa_d \kappa_l}{\kappa_\lambda} \eta_6, & m_{\nu_\tau} &\simeq 0 \end{aligned} \quad (16)$$

The 2×2 mass matrix corresponding to the heavy vector-like fermions in the up sector is given by

$$M_H^U = \frac{\overline{U}_R}{\overline{U}'_R} \begin{pmatrix} \eta_L \eta_1 \left(1 + \frac{\kappa_r^2}{2\kappa_\lambda^2}\right) + \frac{\kappa_u^2}{2\kappa_\lambda^2} \frac{\eta_3^2}{\eta_1 \eta_L} & \frac{\kappa_u \kappa_r}{2\kappa_\lambda^2} \eta_3 \left(\frac{\eta_L \eta_1}{\eta_2} + \frac{\eta_2}{\eta_1 \eta_L}\right) \\ \frac{\kappa_u \kappa_r}{\kappa_\lambda^2} \eta_3 & \eta_2 \left(1 + \frac{\kappa_r^2}{2\kappa_\lambda^2}\right) + \frac{\kappa_u^2}{2\kappa_\lambda^2} \frac{\eta_3^2}{\eta_2} \end{pmatrix} \eta_c \kappa_\lambda \quad (17)$$

Although the basis vectors in (17) are slightly different from those in (12), we use the same symbols. The analog mass-matrix for the down sector of

the heavy fermions is obtained from the above by the replacement ($\eta_3 \rightarrow \eta_4$, $\eta_2 \rightarrow \eta_3$, $\kappa_u \rightarrow \kappa_d$). The charged lepton matrix is obtained from eq. (17) by the replacement ($\eta_c \rightarrow 1$, $\kappa_r \rightarrow \kappa_l$, $\kappa_u \rightarrow \kappa_d$, $\eta_1 \rightarrow \eta_5$, $\eta_2 \rightarrow \eta_7$, $\eta_3 \rightarrow \eta_6$). In the neutral lepton sector, the squared mass matrix corresponding to the (N_R, N'_R) fields is given by

$$M_{lepton}^0 M_{lepton}^{0\dagger} = \begin{pmatrix} (\kappa_l^2 + \kappa_\lambda^2)\eta_L^2\eta_5^2 & \kappa_l\kappa_u\eta_L\eta_5 \\ \kappa_l\kappa_u\eta_L\eta_5 & \kappa_\lambda^2 + \kappa_u^2 \end{pmatrix} \quad (18)$$

The mass eigenvalues of the heavy fermions are obtained from the above:

$$\begin{aligned} M_{U_1} &\simeq \kappa_\lambda\eta_c \left[\eta_L\eta_1\left(1 + \frac{\kappa_r^2}{2\kappa_\lambda^2}\right) + \frac{\eta_3^2}{\eta_1\eta_L} \frac{\kappa_u^2}{2\kappa_\lambda^2} \right] \\ M_{D_1} &\simeq \kappa_\lambda\eta_c \left[\eta_L\eta_1\left(1 + \frac{\kappa_r^2}{2\kappa_\lambda^2}\right) + \frac{\eta_4^2}{\eta_1\eta_L} \frac{\kappa_d^2}{2\kappa_\lambda^2} \right] \\ M_{U_2} &\simeq \kappa_\lambda\eta_c \left[\eta_2\left(1 + \frac{\kappa_r^2}{2\kappa_\lambda^2}\right) + \frac{\eta_3^2}{\eta_2} \frac{\kappa_u^2}{2\kappa_\lambda^2} \right] \\ M_{D_2} &\simeq \kappa_\lambda\eta_c \left[\eta_3\left(1 + \frac{\kappa_r^2}{2\kappa_\lambda^2}\right) + \frac{\eta_4^2}{\eta_3} \frac{\kappa_d^2}{2\kappa_\lambda^2} \right] \\ M_{N_1} &\simeq \kappa_\lambda\eta_L\eta_5 \left[1 + \frac{\kappa_u^2}{2\kappa_\lambda^2} \frac{\eta_L^2\eta_5^2}{\eta_L^2\eta_5^2 - 1} - \frac{\kappa_l^2}{2\kappa_\lambda^2} \frac{1}{\eta_L^2\eta_5^2 - 1} \right] \\ M_{E_1} &\simeq \kappa_\lambda \left[\eta_L\eta_5\left(1 + \frac{\kappa_l^2}{2\kappa_\lambda^2}\right) + \frac{\eta_6^2}{\eta_5\eta_L} \frac{\kappa_d^2}{2\kappa_\lambda^2} \right] \\ M_{N_2} &\simeq \kappa_\lambda \left[1 - \frac{\kappa_u^2}{2\kappa_\lambda^2} \frac{1}{\eta_L^2\eta_5^2 - 1} + \frac{\kappa_l^2}{2\kappa_\lambda^2} \frac{\eta_L^2\eta_5^2}{\eta_L^2\eta_5^2 - 1} \right] \\ M_{E_2} &\simeq \kappa_\lambda \left[\eta_7\left(1 + \frac{\kappa_l^2}{2\kappa_\lambda^2}\right) + \frac{\eta_6^2}{\eta_7} \frac{\kappa_d^2}{2\kappa_\lambda^2} \right] \quad . \end{aligned} \quad (19)$$

It is easy to verify that U_1 and D_1 contain primarily the $SU(2)_L$ -doublet Q -fermions with a small admixture of $SU(2)_R$ -doublets Q' , while U_2 and D_2 contain primarily Q' with a small admixture of Q . Since $M_H^{U,D,E}$ are not symmetric, there will be in general two mixing angles, one for the left-handed sector and one for the right-handed sector. Note, however, that if one neglects the small corrections due to hypercharge interactions $\eta_1 - \eta_7$

and keeps only the lowest order terms in ϵ_L , where $\eta_L \equiv 1 + \epsilon_L$, the above matrices become symmetrical. In this approximation (which should be close to the true scenario, since the neglected terms are only of order 5% or so), the (symmetric) mixing angles for the charged fermion sectors are given by

$$\begin{aligned}\tan 2\theta_{U,D} &\simeq \frac{2\kappa_{u,d}\kappa_r}{\kappa_\lambda^2(\eta_L - 1)} \\ \tan 2\theta_E &\simeq \frac{2\kappa_d\kappa_l}{\kappa_\lambda^2(\eta_L - 1)}.\end{aligned}\quad (20)$$

The mass eigenstates $(U_1)_{L,R}$ and $(U_2)_{L,R}$ are given by

$$\begin{aligned}(U_1)_{L,R} &= \cos\theta_U U_{L,R} + \sin\theta_U U'_{L,R} \\ (U_2)_{L,R} &= -\sin\theta_U U_{L,R} + \cos\theta_U U'_{L,R}\end{aligned}\quad (21)$$

and likewise $(D_{1,2})_{L,R}$ and $(E_{1,2})_{L,R}$.

Note that the mixing in the up-sector of the heavy quarks denoted by θ_U is proportional to κ_u and hence is large, while in the down sector, the mixing angle θ_D and θ_E are proportional to κ_d ; they are consequently small (since $\kappa_d/\kappa_u \approx m_b/m_t \ll 1$).

The mixing angle for the right-handed neutral leptons (N_R, N'_R) is

$$\tan 2\theta_N^R \simeq \frac{2\kappa_u\kappa_l\eta_L}{\kappa_\lambda^2(\eta_L^2 - 1)}.\quad (22)$$

In the left-handed neutral lepton sector, the mixing is somewhat more complicated, due to the fact that $\nu_{\tau_L} - N_L$ mixing is not negligible. Starting from eq. (15), we obtain the exact orthogonal matrix V_L^ν which transforms the gauge eigenstate (ν_L^g) to the mass eigenstate via $\nu_L^m = V_L^\nu \nu_L^g$, where

$$V_L^\nu = \begin{pmatrix} c_1 & -c_2 s_1 & -s_1 s_2 \\ c_3 s_1 & c_1 c_2 c_3 + s_2 s_3 & c_1 c_3 s_2 - c_2 s_3 \\ -s_1 s_3 & -c_1 c_2 s_3 + c_3 s_2 & -c_2 c_3 - c_1 s_2 s_3 \end{pmatrix}.\quad (23)$$

Here $s_i = \sin\theta_i, c_i = \cos\theta_i$ with

$$\begin{aligned}
\tan\theta_1 &= \frac{\sqrt{\kappa_u^2 + \kappa_l^2}}{\kappa_\lambda}, \quad \tan\theta_2 = \frac{\kappa_u}{\kappa_l} \\
\tan 2\theta_3 &= \frac{2\kappa_u\kappa_l\kappa_\lambda\sqrt{\kappa_\lambda^2 + \kappa_u^2 + \kappa_l^2}(\eta_L^2\eta_5^2 - 1)}{(\kappa_u^2 + \kappa_l^2\eta_L^2\eta_5^2)(\kappa_l^2 + \kappa_u^2 + \kappa_\lambda^2) - \kappa_\lambda^2(\kappa_l^2 + \eta_L^2\eta_5^2\kappa_u^2)} \\
&\simeq \frac{2\kappa_u\kappa_l}{\kappa_l^2 - \kappa_u^2}.
\end{aligned} \tag{24}$$

Since $\kappa_{u,l} \ll \kappa_\lambda$, the mixing angle θ_1 is small ($O(1/5$ to $1/10$), say); but θ_2 and θ_3 can be relatively larger, since κ_u and κ_l are expected to be comparable within a factor of two (say). Note that $\theta_2 \simeq \theta_3$.

To have a feel for the numerical values of the masses and the mixing angles that are relevant to the heavy families, we need to know the effective parameters which enter into the fermion mass-matrix. Talking of these, note that the full 5×5 mass-matrices (eq. (2)) of the four sectors – i.e., u, d, l and ν – which in general could involve 100 parameters, even if they are all real, are essentially determined (barring electroweak corrections and $m_{\text{dir}}^{(0)}$ which are important only for the light families) by just six effective parameters: $p, \kappa_u, \kappa_d, \kappa_r, \kappa_l$ and κ_λ . These successfully describe the gross features of the masses of the known fermions – in particular their magnitudes and the inter-family hierarchy [3]. In fact, even these few parameters are not completely arbitrary (unlike in the case of an elementary Higgs-picture) in that we know apriori their approximate values to within a factor of 10, say. For example, as mentioned above [1,3], we expect that $\kappa_\lambda = \mathcal{O}(a_\lambda \Lambda_M (\Lambda_M/M_{Pl})) \sim \mathcal{O}(1 \text{ TeV})$ and that $(\kappa_{f,c}/\kappa_\lambda) \approx 1/3$ to $1/10$, while $1/10 < p < 1/2$ (say).

Guided in part by the expected order of magnitude of these parameters and their ratios as mentioned above and by the observed masses of the muon and the tau-families, we are led to the following values for certain ratios of

these parameters:

$$(\kappa_r/\kappa_l) \approx 0.9 \text{ to } 1 \quad (25a)$$

$$(\kappa_d/\kappa_u) \approx \frac{1}{40 \pm 10} \quad (25b)$$

$$(\kappa_l/\kappa_\lambda) \approx 1/3 \quad (25c)$$

$$\frac{\kappa_u \kappa_r}{\kappa_\lambda} \approx (26 \text{ to } 33) \text{ GeV} \quad (25d)$$

These values are arrived at as follows [See Ref. 3, a more detailed discussion will be presented elsewhere]. The ratio (κ_r/κ_l) is obtained from (m_b/m_τ) including QCD and electroweak renormalization effects, and (κ_d/κ_u) is obtained from (m_b/m_t) (see below). The ratio $(\kappa_l/\kappa_\lambda)$ is obtained utilizing corrections of order $(\kappa_l/\kappa_\lambda)^2$ to the leading see-saw contribution to m_τ (see Eq. (7) of Ref. 3) and optimizing the choice of parameters consistent with their expected ranges so as to obtain a reasonable fit to (m_μ/m_τ) within 10%, with the inclusion of electroweak corrections [Ref. 3]. Finally, the ratio $(\kappa_u \kappa_r/\kappa_\lambda)$ given by (25-d) is obtained by using the fact that internal consistency of the model requires $m_t(\text{phys}) \lesssim 160 \text{ GeV}$ [3,13].

Using (25-a) and (25-c) we obtain

$$(\kappa_r/\kappa_\lambda) \approx 0.33 \quad (25e)$$

which, combined with (25-d), yields $\kappa_u \approx (76 - 100) \text{ GeV}$. Allowing for some uncertainty in $(\kappa_l/\kappa_\lambda)$ which affects κ_u , we take

$$\kappa_u \approx (40 - 100) \text{ GeV} \quad (25f)$$

This still leaves κ_λ undetermined (although its order of magnitude is known). We observe that the “number” of light neutrinos measured at LEP places an upper limit on $(\kappa_u/\kappa_\lambda)$. Owing to $\nu_\tau - N'$ mixing, the model yields [6]: $N_\nu = 2 + [1 - 2(\kappa_u/\kappa_\lambda)^2]$. Comparing with the observed value [10]

$N_\nu = 2.99 \pm 0.03$, we get $(\kappa_u/\kappa_\lambda) \lesssim 1/7$ for $N_\nu \gtrsim 2.96$. Allowing for agreement within two standard deviations on the one hand and following our general expectations for $(\kappa_u/\kappa_\lambda)$ mentioned above, on the other hand, we take:

$$(\kappa_u/\kappa_\lambda) \approx 1/5 \text{ to } 1/10 . \quad (25g)$$

Values of $(\kappa_u/\kappa_\lambda)$ near $1/10$ can be probed if N_ν can be measured with an accuracy of 0.01 to 0.02. For the present, using (25-f and g), we get

$$\begin{aligned} \kappa_\lambda &\approx (1 \text{ to } 2)(200 - 500) \text{ GeV} \\ &\approx (200 \text{ GeV} - 1 \text{ TeV}) . \end{aligned} \quad (25h)$$

To have a feel for the masses and mixing angles, consider a representative set of values:

$$\begin{aligned} \kappa_\lambda &\approx 575 \text{ GeV}, \quad \kappa_u/\kappa_\lambda = 1/6, \quad \kappa_d/\kappa_u \simeq 1/40 \\ \kappa_r/\kappa_\lambda &\approx 1/3, \quad \kappa_l/\kappa_\lambda \approx 1/3 \end{aligned} \quad (26)$$

This yields [9,13]

$$\begin{aligned} (m_t, m_b, m_\tau)_{1.5 \text{ TeV}} &\simeq (135, 3.4, 1.5) \text{ GeV} \\ (m_t, m_b, m_\tau)_{phys} &\simeq (157, 4.7, 1.7) \text{ GeV} \end{aligned} \quad (27)$$

$$\begin{aligned} U_1 &\simeq 1814 \text{ GeV} & N_1 &\simeq 765 \text{ GeV} \\ D_1 &\simeq 1787 \text{ GeV} & E_1 &\simeq 763 \text{ GeV} \\ U_2 &\simeq 1504 \text{ GeV} & N_2 &\simeq 581 \text{ GeV} \\ D_2 &\simeq 1465 \text{ GeV} & E_2 &\simeq 667 \text{ GeV} \end{aligned} \quad (28)$$

$$\begin{aligned} \theta_U &\simeq 1/5, \quad \theta_D \simeq 1/210, \quad \theta_E \simeq 1/140 \\ \theta_N^R &\simeq 1/10.6, \quad \tan\theta_1 \simeq 1/2.7, \quad \tan\theta_2 \simeq 1/2, \quad \tan\theta_3 \simeq 1/2.6 . \end{aligned} \quad (29)$$

There are corrections of order 10% to these numbers arising from the mixing of the top and vector-like families with the lighter e and μ families, but they can be neglected for our present purpose.

While the precise values of the masses and the mixing angles depend upon the specific choice of parameters given by (26), which in general could vary within the range indicated in (25 a-h), a few qualitative features would still remain which are worth noting:

(1) It is clear that the mixing angles θ_D and θ_E are tiny ($\sim 10^{-2}$) while those in the up-sector, including neutrino-members, are appreciable. This is simply because $\kappa_d/\kappa_u \sim 1/40$ and thus $\kappa_d/\kappa_\lambda \lesssim 1/240$.

(2) Given that $\theta_D \ll 1$ and $\theta_U \approx 1/5$, it follows from (21) that D_1 and even U_1 are mostly composed of Q -fermions which are $SU(2)_L$ -doublets while U_2 and D_2 are mostly composed of Q' -fermions which are $SU(2)_R$ -doublets. This is important for their decay modes.

(3) Note (from (19) and (28)) that the pair U_1 and D_1 are nearly degenerate to within about 10-30 GeV, so also the pair N_1 and E_1 , and to a lesser extent the pair N_2 and E_2 . But the (U_1, D_1) pair is substantially heavier, by about a few hundred GeV, than the pair (U_2, D_2) . Similarly the (N_1, E_1) pair is heavier by about 100 GeV than the pair (N_2, E_2) . This is because the (U_1, D_1) and also the (N_1, E_1) pair receive enhancement due to $SU(2)_L$ -renormalization factor η_L , which is, however, absent for the (U_2, D_2) and (N_2, E_2) -pairs.

The mass-gap between the quark-pairs (U_1, D_1) versus (U_2, D_2) is much larger than that between the leptonic pairs (N_1, E_1) versus (N_2, E_2) because the QCD-enhancement factor $\eta_c \approx 2.39$ multiplies $\eta_L \approx 1.23$ in the case of quarks, but not for the leptons.

All these qualitative features of the heavy fermion mass spectrum remain

intact even if the relevant parameters are varied within the range indicated in eq. (25).

(4) Given this mass-pattern, we see that $U_1 \rightarrow D_1 + W$ and likewise $N_1 \rightarrow E_1 + W$ are forbidden kinematically, while decays such as $U_1 \rightarrow D_2 + W$, $U_1 \rightarrow U_2 + Z$, $D_1 \rightarrow U_2 + W$, $D_1 \rightarrow D_2 + Z$ and possibly $U_2 \rightarrow D_2 + W$ are kinematically allowed.

III. Coupling and Decay Modes of the Heavy Fermions

In terms of the mixing angles in the heavy sector, the coupling of the W_L^\pm and Z^0 to the left-handed as well as the right-handed charged fermions can be written down. We list them in Tables 1-5 presented below.

Table. 1. The coupling of W_L^\pm to the left-chiral quarks of the τ and the two vector-like families. An overall factor $(g/\sqrt{2})\gamma_\mu$ is not displayed, but should be understood.

	\bar{t}_L	\bar{U}_{1L}	\bar{U}_{2L}
b_L	$1 - (\kappa_u^2 + \kappa_d^2)/(2\kappa_\lambda^2)$	$-(\kappa_u/\kappa_\lambda)\sin\theta_U$	$(\kappa_u/\kappa_\lambda)\cos\theta_U$
D_{1L}	$-(\kappa_d/\kappa_\lambda)\sin\theta_D$	$\cos\theta_U\cos\theta_D$	$\sin\theta_U\cos\theta_D$
D_{2L}	$(\kappa_d/\kappa_\lambda)\cos\theta_D$	$\cos\theta_U\sin\theta_D$	$\sin\theta_U\sin\theta_D$

Table. 2. The coupling of W_L^\pm to the right-chiral quarks of the tau and the vector-like families. An overall factor $(g/\sqrt{2})\gamma_\mu$ is not displayed.

	\bar{t}_R	\bar{U}_{1R}	\bar{U}_{2R}
b_R	$\kappa_u\kappa_d/(\kappa_\lambda^2\eta_L^2)$	$-(\kappa_d/\kappa_\lambda\eta_L)\cos\theta_U$	$-(\kappa_d/\kappa_\lambda\eta_L)\sin\theta_U$
D_{1R}	$-(\kappa_u/\kappa_\lambda\eta_L)\cos\theta_D$	$\cos\theta_U\cos\theta_D$	$\sin\theta_U\cos\theta_D$
D_{2R}	$-(\kappa_r/\kappa_\lambda)\sin\theta_D$	$\cos\theta_U\sin\theta_D$	$\sin\theta_U\sin\theta_D$

Table. 3. The coupling of Z^0 to the left-chiral quarks and charged leptons collectively denoted by q_f and Q_f , respectively, where $i = 1, 2$, $f = \text{up, down}$,

charged lepton. An overall factor $-(g/\cos\theta_W)\gamma_\mu$ is not displayed. Here $T_3 = (1/2, -1/2, -1/2)$ and $Q = (2/3, -1/3, -1)$ for $f = u, d, l$, $s_W^2 = \sin^2\theta_W$ and $F = U, D, E, N$.

	\bar{q}_L^f	\bar{Q}_{1L}^f	\bar{Q}_{2L}^f
q_L^f	$T_3(1 - \kappa_f^2/\kappa_\lambda^2) - Qs_W^2$	$-T_3(\kappa_f/\kappa_\lambda)\sin\theta_F$	$T_3(\kappa_f/\kappa_\lambda)\cos\theta_F$
Q_{1L}^f	$-T_3(\kappa_f/\kappa_\lambda)\sin\theta_F$	$T_3\cos^2\theta_F - Qs_W^2$	$T_3\cos\theta_F\sin\theta_F$
Q_{2L}^f	$T_3(\kappa_f/\kappa_\lambda)\cos\theta_F$	$T_3\cos\theta_F\sin\theta_F$	$T_3\sin^2\theta_F - Qs_W^2$

Table. 4. The coupling of Z^0 to the right-chiral quarks and charged leptons with the same notation as in Table 3.

	\bar{q}_R^f	\bar{Q}_{1R}^f	\bar{Q}_{2R}^f
q_R^f	$T_3(\kappa_f^2/\kappa_\lambda^2\eta_L^2) - Qs_W^2$	$-T_3(\kappa_f/\kappa_\lambda\eta_L)\cos\theta_F$	$-T_3(\kappa_f/\kappa_\lambda)\sin\theta_F$
Q_{1R}^f	$-T_3(\kappa_f/\kappa_\lambda\eta_L)\cos\theta_F$	$T_3\cos^2\theta_F - Qs_W^2$	$T_3\cos\theta_F\sin\theta_F$
Q_{2R}^f	$-T_3(\kappa_f/\kappa_\lambda)\sin\theta_F$	$T_3\cos\theta_F\sin\theta_F$	$T_3\sin^2\theta_F - Qs_W^2$

Table. 5. The coupling of Z^0 to the left-chiral neutral leptons. Notation same as in Table 3. The overall factor $-(g/\cos\theta_W)\gamma_\mu$ is not displayed.

	$\bar{\nu}_{\tau L}$	\bar{N}_{1L}	\bar{N}_{2L}
$\nu_{\tau L}$	$1 - s_1^2 s_2^2$	$s_1 s_2 (c_1 c_3 s_2 - c_2 s_3)$	$-s_1 s_2 (c_2 c_3 + c_1 s_2 s_3)$
N_{1L}	$s_1 s_2 (c_1 c_3 s_2 - c_2 s_3)$	$1 - (c_1 c_3 s_2 - c_2 s_3)^2$	$(c_1 c_3 s_2 - c_2 s_3) (c_2 c_3 + c_1 s_2 s_3)$
N_{2L}	$-s_1 s_2 (c_2 c_3 + c_1 s_2 s_3)$	$(c_1 c_3 s_2 - c_2 s_3) (c_2 c_3 + c_1 s_2 s_3)$	$1 - (c_2 c_3 + c_1 s_2 s_3)^2$

The coupling of W_L^+ to the left-handed charged lepton currents take the following form:

$$\begin{aligned}
 \bar{\nu}_\tau \tau_L &: 1 - \frac{\kappa_u^2 + \kappa_d^2}{2\kappa_\lambda^2} \\
 \bar{\nu}_\tau E_{1L} &: -\frac{\kappa_d}{\kappa_\lambda} \sin\theta_E \\
 \bar{\nu}_\tau E_{2L} &: \frac{\kappa_d}{\kappa_\lambda} \cos\theta_E
 \end{aligned}$$

$$\begin{aligned}
\overline{N}_{1L}\tau_L &: c_3s_1 - (c_2c_3 + s_2s_3)\frac{\kappa_l}{\kappa_\lambda} \\
\overline{N}_{1L}E_{1L} &: c_3s_1\left(\frac{\kappa_l}{\kappa_\lambda}\cos\theta_E - \frac{\kappa_d}{\kappa_\lambda}\sin\theta_E\right) + (c_1c_2c_3 + s_2s_3)\left(\cos\theta_E(1 - \frac{\kappa_l^2}{2\kappa_\lambda^2}) + \sin\theta_E\frac{\kappa_l\kappa_d}{2\kappa_\lambda^2}\right) \\
\overline{N}_{1L}E_{2L} &: c_3s_1\left(\frac{\kappa_l}{\kappa_\lambda}\sin\theta_E + \frac{\kappa_d}{\kappa_\lambda}\cos\theta_E\right) + (c_1c_2c_3 + s_2s_3)\left(\sin\theta_E(1 - \frac{\kappa_l^2}{2\kappa_\lambda^2}) - \cos\theta_E\frac{\kappa_l\kappa_d}{2\kappa_\lambda^2}\right) \\
\overline{N}_{2L}\tau_L &: -s_1s_3 - (-c_1c_2s_3 + c_3s_2)\frac{\kappa_l}{\kappa_\lambda} \\
\overline{N}_{2L}E_{1L} &: -s_1s_3\left(\frac{\kappa_l}{\kappa_\lambda}\cos\theta_E - \frac{\kappa_d}{\kappa_\lambda}\sin\theta_E\right) + (-c_1c_2s_3 + c_3s_2)\left(\cos\theta_E(1 - \frac{\kappa_l^2}{2\kappa_\lambda^2}) + \sin\theta_E\frac{\kappa_l\kappa_d}{2\kappa_\lambda^2}\right) \\
\overline{N}_{2L}E_{2L} &: -s_1s_3\left(\frac{\kappa_l}{\kappa_\lambda}\sin\theta_E + \frac{\kappa_d}{\kappa_\lambda}\cos\theta_E\right) + (-c_1c_2s_3 + c_3s_2)\left(\sin\theta_E(1 - \frac{\kappa_l^2}{2\kappa_\lambda^2}) - \cos\theta_E\frac{\kappa_l\kappa_d}{2\kappa_\lambda^2}\right)
\end{aligned} \tag{30}$$

Using Tables 1-5 and the mixing angles listed, the following pattern of decay modes for the heavy fermions emerge (Table 6). We list all dominant modes and only some suppressed or forbidden ones. The reason for suppression or dominance of a mode can be inferred by looking at the third column. Rates which are proportional to $\sin^2\theta_D$ and/or $(\kappa_d/\kappa_\lambda)^2$ are suppressed.

Table. 6. Pattern of heavy fermion decay modes. We have approximated the leptonic mixing angles $\theta_{1,2,3}$ of Eq. (24) by keeping only the lowest order terms in κ_l/κ_λ and κ_u/κ_l .

Particle	Decay Modes	Rate (\propto)	Comment
U_1	$\rightarrow D_1 + W^+$ $\rightarrow D_2 + W^+$ $\rightarrow U_2 + Z$ $\rightarrow t_L + Z$ $\rightarrow t_R + Z$	Kin. Forbidden $(\cos\theta_U \sin\theta_D)^2$ $(\cos\theta_U \sin\theta_U)^2$ $((\kappa_u/2\kappa_\lambda) \sin\theta_U)^2$ $((\kappa_u/2\kappa_\lambda \eta_L) \cos\theta_U)^2$	Suppressed Dominant Appreciable Appreciable
U_2	$\rightarrow D_1 + W^+$ $\rightarrow b_L + W^+$ $\rightarrow t_L + Z$ $\rightarrow t_R + Z$	Kin. Forbidden $((\kappa_u/\kappa_\lambda) \cos\theta_U)^2$ $((\kappa_u/2\kappa_\lambda) \cos\theta_U)^2$ $((\kappa_u/2\kappa_\lambda) \sin\theta_U)^2$	Dominant Appreciable Appreciable
D_1	$\rightarrow U_2 + W^-$ $\rightarrow t_L + W^-$ $\rightarrow t_R + W^-$	$(\sin\theta_U \cos\theta_D)^2$ $((\kappa_d/\kappa_\lambda) \sin\theta_D)^2$ $((\kappa_u/\kappa_\lambda \eta_L) \cos\theta_D)^2$	Dominant-1 Highly suppressed Dominant-2
D_2	$\rightarrow t_L + W^-$ $\rightarrow t_R + W^-$ $\rightarrow b_L + Z$ $\rightarrow b_R + Z$	$((\kappa_d/\kappa_\lambda) \cos\theta_D)^2$ $((\kappa_r/\kappa_\lambda) \sin\theta_D)^2$ $((\kappa_d/2\kappa_\lambda) \cos\theta_D)^2$ $((\kappa_d/2\kappa_\lambda) \sin\theta_D)^2$	Suppressed Dominant Suppressed Suppressed
N_1	$\rightarrow E_1 + W$ $\rightarrow E_{2L} + W$ $\rightarrow N_{2L} + Z$ $\rightarrow N_{2R} + Z$ $\rightarrow \nu_\tau + Z$ $\rightarrow \tau_L + W$	Kin. Forbidden $(\kappa_l \kappa_d / \kappa_\lambda^2 (\eta_L - 1))^2$ $(\kappa_u \kappa_l / (2\kappa_\lambda^2))^2$ $((1/2) \cos\theta_U \sin\theta_U)^2$ $((\kappa_u/\kappa_\lambda) (\kappa_u^3/\kappa_l^3 - \kappa_u \kappa_l / 2\kappa_\lambda^2))^2$ $(\kappa_l/\kappa_\lambda)^2$	Highly suppressed Phase space supp. Phase space supp. Dominant Dominant
N_2	$\rightarrow E_1 + W$ $\rightarrow \tau_L + W$ $\rightarrow \nu_\tau + Z$	Kin. Forbidden $(\kappa_u/\kappa_\lambda)^2$ $(\kappa_u/\kappa_\lambda)^2$	Dominant Dominant
E_1	$\rightarrow \tau_R + Z$ $\rightarrow \tau_L + Z$	$((\kappa_d/2\kappa_\lambda \eta_L) \cos\theta_E)^2$ $((\kappa_d/2\kappa_\lambda) \sin\theta_E)^2$	Supp. rate, but dom. mode Doubly suppr.
E_2	$\rightarrow \nu_\tau + W$ $\rightarrow \tau_L + Z$ $\rightarrow \tau_R + Z$	$((\kappa_d/\kappa_\lambda) \cos\theta_E)^2$ $((\kappa_d/2\kappa_\lambda) \cos\theta_E)^2$ $((\kappa_d/2\kappa_\lambda) \sin\theta_E)^2$	Suppr. rate but dom. mode Same Highly suppr.

In above, it is to be understood that for modes, where chirality is not

shown (e.g., $U_1 \rightarrow D_1 + W^+$), either chirality has the same amplitude. It is interesting to note, however, that *there are modes for which there is a strong preference for either the left or the right chirality*. For example, $D_1 \rightarrow t_L + W^-$ is highly suppressed, but $D_1 \rightarrow t_R + W^-$ would have appreciable branching ratio. Likewise, $D_2 \rightarrow t_R + W^-$ is dominant, but $D_2 \rightarrow t_L + W^-$ is suppressed. Furthermore, $E_1 \rightarrow \tau_R + Z$ is dominant but $E_1 \rightarrow \tau_L + Z$ is suppressed, while $E_2 \rightarrow \tau_L + Z$ is dominant and $E_2 \rightarrow \tau_R + Z$ is suppressed. Such interesting decay patterns and the correlation with chirality are clearly features that are intimately tied as much to the nature of the gauge couplings of the canonical fields $Q_{L,R}$ and $Q'_{L,R}$ as to the nature of the fermion mass-matrix given by (2) and (3). In this sense, they are truly distinguishing features of the model.

IV. Production and Signals

Production of the heavy quarks in pairs by hadronic colliders at SSC and LHC energies has been studied in a number of papers [14]. These studies typically yield production cross sections at $\sqrt{s} = 40 \text{ TeV}$ as follows:

Table. 7. Heavy quark pair production cross section at $\sqrt{s} = 40 \text{ TeV}$ for $U\bar{U}$. The cross section σ listed is in nb .

m_U	3 TeV	2.4 TeV	2 TeV	1.5 TeV	1 TeV	500 GeV
$\sigma(pp \rightarrow U\bar{U})$	2×10^{-6}	10^{-5}	5×10^{-5}	3×10^{-4}	2.5×10^{-3}	5×10^{-2}

Assuming that a future version of the SSC will be built one day in the near future, the production cross section noted above would lead to about 2.5×10^4 events per year for $m_U = 1 \text{ TeV}$, with a luminosity of $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$.

We now consider the likely signals of pair production of such heavy quarks by considering their expected decay modes.

(1)

$$\begin{aligned}
pp, p\bar{p} &\rightarrow U_1 + \bar{U}_1 \\
U_1 &\rightarrow U_2 + Z; (U_2 \rightarrow \{b + W, \text{or}, t + Z\}; t \rightarrow b + W) \\
\bar{U}_1 &\rightarrow \bar{U}_2 + Z; (\bar{U}_2 \rightarrow (\{\bar{b} + W, \text{or}, \bar{t} + Z\}; \bar{t} \rightarrow \bar{b} + W))
\end{aligned} \quad (31)$$

Thus

$$(pp, \bar{p}p) \rightarrow U_1 + \bar{U}_1 \rightarrow 2Z + 2W + b\bar{b} \quad (32a)$$

$$\text{or } \rightarrow 4Z + 2W + b\bar{b} \quad (32b)$$

$$\text{or } \rightarrow 3Z + 2W + b\bar{b} \quad (32c)$$

(2)

$$\begin{aligned}
pp, \bar{p}p &\rightarrow U_2 + \bar{U}_2 \\
U_2 &\rightarrow \{b + W, \text{or}, t + Z\}; t \rightarrow b + W \\
\bar{U}_2 &\rightarrow \{\bar{b} + W, \text{or}, \bar{t} + Z\}; \bar{t} \rightarrow \bar{b} + W
\end{aligned} \quad (33)$$

Thus,

$$(pp, \bar{p}p) \rightarrow U_2 + \bar{U}_2 \rightarrow 2W + b\bar{b} \quad (34a)$$

$$\text{or } \rightarrow 2W + 2Z + b\bar{b} \quad (34b)$$

$$\text{or } \rightarrow 2W + Z + b\bar{b} \quad (34c)$$

(3)

$$\begin{aligned}
pp, \bar{p}p &\rightarrow D_1 + \bar{D}_1 \\
D_1 &\rightarrow \{U_2 + W^-, \text{or}, t + W^-\}; t \rightarrow b + W \\
\bar{D}_1 &\rightarrow \{\bar{U}_2 + W, \text{or}, \bar{t} + W\}; \bar{t} \rightarrow \bar{b} + W
\end{aligned} \quad (35)$$

Thus noting signals for $U_2\overline{U}_2$ in (33), we expect

$$(pp, \bar{p}p) \rightarrow D_1\overline{D}_1 \rightarrow 4W + b\bar{b} \quad (36a)$$

$$\text{or } \rightarrow 4W + 2Z + b\bar{b} \quad (36b)$$

$$\text{or } \rightarrow 4W + Z + b\bar{b} \quad (36c)$$

(4)

$$\begin{aligned} (pp, \bar{p}p) &\rightarrow D_2 + \overline{D}_2 \\ D_2 &\rightarrow t + W, \text{ or } t \rightarrow b + W \\ \overline{D}_2 &\rightarrow \bar{t} + W, \text{ or, } \bar{t} \rightarrow \bar{b} + W \end{aligned} \quad (37)$$

$$(pp, \bar{p}p) \rightarrow D_2\overline{D}_2 \rightarrow 4W + b\bar{b} \quad (38)$$

Before discussing the signals for heavy lepton pair-production, we see already from (31-38) that pair-production of the heavy quarks of the model would lead to distinctive signatures – such as production of $4Z + W^+W^- + b\bar{b}$ – which would not be confused first of all with the background expected from the standard model involving the three families and the Higgs boson and second with the signals expected from pair-production of some general heavy quark belonging, for example, to a standard fourth family.

Heavy lepton pair production in hadronic and leptonic machines will proceed through virtual photon and Z^0 -productions. These cross sections have been studied in Ref [14]. The prominent decay modes of the heavy leptons are listed in Table [6], from which we arrive at the signals for heavy lepton production in the model.

(1)

$$\begin{aligned} (p\bar{p}, e^+e^-) &\rightarrow N_1 + \overline{N}_1 \\ N_1 &\rightarrow \{\nu_\tau + Z, \text{ or, } \tau + W\}; \quad \tau \rightarrow e^- + \bar{\nu} \end{aligned}$$

$$\overline{N}_1 \rightarrow \{\overline{\nu}_\tau + Z, \text{or, } \overline{\tau} + W\}; \quad \overline{\tau} \rightarrow e^+ + \nu \quad (39)$$

$$p\overline{p} \rightarrow N_1 \overline{N}_1 \rightarrow 2Z + \nu_\tau + \overline{\nu}_\tau \quad (40a)$$

$$\text{or } \rightarrow 2W + \tau\overline{\tau} \quad (40b)$$

$$\text{or } \rightarrow Z + W + \overline{\tau} + \nu_\tau \quad (40c)$$

(2)

$$(p\overline{p}, e^+ e^-) \rightarrow N_2 + \overline{N}_2$$

$$N_2 \rightarrow \{\tau + W, \text{or, } \nu_\tau + Z\} \quad (41)$$

$$(p\overline{p}, e^+ e^-) \rightarrow \{2Z + \nu_\tau + \overline{\nu}_\tau; 2W + \tau + \overline{\tau}; W + Z + \overline{\tau} + \nu_\tau\} \quad (42)$$

(3)

$$(pp, e^+ e^-) \rightarrow E_1 + \overline{E}_1$$

$$E_1 \rightarrow \tau_R + Z; \quad \overline{E}_1 \rightarrow \overline{\tau}_R + Z$$

$$(p\overline{p}, e^+ e^-) \rightarrow 2Z + \tau_R + \overline{\tau}_R \quad (43)$$

(4)

$$(p\overline{p}, e^+ e^-) \rightarrow E_2 + \overline{E}_2$$

$$E_2 \rightarrow \{\nu_\tau + W; \tau_L + Z\}; \quad \overline{E}_2 \rightarrow \{\overline{\nu}_\tau + W; \overline{\tau}_L + Z\}$$

$$\rightarrow \{2W + \nu_\tau + \overline{\nu}_\tau; 2Z + \tau_L + \overline{\tau}_L; W + Z + \nu_\tau + \overline{\tau}_L\} \quad (44)$$

Thus $N_1 \overline{N}_1$, $N_2 \overline{N}_2$ and $E_2 \overline{E}_2$ lead essentially to the same signals with three possible channels, while $E_1 \overline{E}_1$ leads only to $2Z + \tau_R + \overline{\tau}_R$. These may be compared with standard model signals arising from $pp, p\overline{p}, e^+ e^- \rightarrow H + Z \rightarrow ZZZ \rightarrow \tau^+ \tau^- + 2Z$. One distinction which may not be easy to

utilize in practice is that the SM would yield $\tau^+\tau^-$ with nearly equal left and right helicities for $\sin^2\theta_W \simeq 1/4$ (since $J_\mu^Z \propto J_\mu^{3em} - \sin^2\theta_W$), whereas the model discussed here [1-3] yields predominantly $\tau_R\bar{\tau}_R$. The other distinction has to do with expected rates and kinematics.

One additional feature mentioned in Chapter I is that one expects significant production of a single heavy lepton (N_1 or N_2) together with ν_τ in an e^+e^- machine of appropriate energy through a virtual Z (see Table 5 and eqs. (23), (24) and (29) for relevant amplitudes and mixing angles) followed by the dominant decay of N_1 (or N_2) into $(\nu_\tau + Z) \rightarrow \nu_\tau + e^+e^-$;

$$\begin{aligned} e^+e^- &\rightarrow "Z" \rightarrow N_1 \text{ (or } N_2) + \nu_{\tau_L} \\ &\rightarrow (Z + \nu_{\tau_L}) + \nu_{\tau_L} \rightarrow (e^+e^-)_{Z-mass} + \text{ missing energy} \end{aligned} \quad (45)$$

The amplitude for $"Z" \rightarrow N_{2L} + \nu_{\tau L}$ (for example) is $\sim s_1 s_2 \sim (1/6 \text{ to } 1/10)$ relative to $"Z" \rightarrow e^+e^-$. Thus the rates for such spectacular signals, characterized by eq. (45) would be appreciable and observable.

V. Vector-Like Fermions in Other Models and Concluding Remarks

Before concluding we wish to note that vector-like fermions have been introduced in other contexts by several authors [15-18]. The case considered here [1-3] possesses, however, some distinguishing features as regards the origin and the nature of the vector-like families compared to those arising in the context of other works [15-18]. Below we present some of these distinctions.

In Refs. (15) and (16), $SU(2)_L$ -singlet vector-like families, which are in part analogous to our $Q'_{L,R} = (U', D', N', E')_{L,R}$, have been introduced to generate, with the choice of suitable Higgs multiplets and/or discrete

symmetries, a see-saw like fermion mass-matrix. A second motivation to introduce vector-like families has been to account for the vanishing of the strong CP $\overline{\theta}$ -parameter at the tree-level [17]. A related attempt introduces $SU(2)_L$ -doublet vector-like quarks, but not the corresponding leptons and the analog of $SU(2)_L$ -singlet $Q'_{L,R}$, together with an extra $U(1)$ -gauge symmetry to arrange so that $\overline{\theta}$ is zero at the tree-level. The similarities and the differences between the vector-like fermions arising in these models and those arising in the present case [1-3] are listed below.

- (i) First, the vector-like fermions or families are not predicted in any of these models [15-18] by an apriori theoretical reason, based on some higher symmetry or other grounds, whereas in the present case, they arise in a compelling manner as a general consequence of SUSY-compositeness in a class of SUSY theories based on QCD-like binding force [1-3].
- (ii) Second, the masses of the vector-like families, which are $SU(2)_L \times U(1)$ -invariant, are not protected by a symmetry in the models of Ref. [15-18]. They could apriori be as large as the grand unification or even Planck scale and thus inaccessible to future accelerators. By contrast, for the case considered here, the masses of the vector-like families are protected by the non-anomalous R symmetry, which is broken by the desired amount only by the SUSY-breaking metagaugino condensate $\langle \lambda \lambda \rangle$. Thus their masses are protected to the same extent as supersymmetry breaking; both are damped by the Planck scale and are naturally of order 1 TeV [1,7,3]. This is what makes them accessible to accelerators.
- (iii) Third, invariably the other models, as they stand, contain only $SU(2)_L$ -singlet family [16] or families [15], which are in part analogous to our $SU(2)_R$ -doublet family $Q'_{L,R} = (U', D', N', E')_{L,R}$, but none of them [15-18] contain the analog of our complete $SU(2)_L$ -doublet family $Q_{L,R} = (U, D, N, E)_{L,R}$

together with the $SU(2)_L$ -singlet $Q'_{L,R}$. In short, the coupling of the vector-like fermions to W_L^\pm can serve to distinguish between the cases of interest.

Vector-like fermions arise in a more compelling manner for the case of a **27** of E_6 , which can arise from the heterotic superstring theory. The **27** splits into **(16 + 10 + 1)** under $SO(10)$. The **16** contains the standard model fermions and a $(\nu_R)^c$. The **1** gives a singlet neutral lepton N_L . The **10** contains a $(2_L, 2_R, 1^c)$ and a $(1_L, 1_R, 6^c)$ under $SU(2)_L \times SU(2)_R \times SU(4)^c$ of $SO(10)$. The $(1, 1, 6^c)$ gives a $SU(3)^c$ -color triplet of quarks, with charge $-1/3$ ($B - L = -2/3$), and an antitriplet with charge $+1/3$, which are *singlets* of $SU(2)_L$, while the $(2, 2, 1)$ gives a pair of leptonic $SU(2)_L$ -doublets, which couple vectorially to $SU(2)_L$ gauge bosons. These also form a pair of $SU(2)_R$ -doublets. These vector-like leptons differ, of course, from the case considered here in that they are not accompanied by vector-like quarks to make a complete family. Furthermore, they couple vectorially to $SU(2)_L$ as well as $SU(2)_R$ gauge bosons.

In summary, two vector-like families, not more not less [19], with one coupling vectorially to W_L 's and the other to W_R 's (before mass-mixing), with masses of order 1 TeV, constitute a *hall-mark* and a crucial prediction of the SUSY preon model [1-3]. *There does not seem to be any other model including superstring-inspired models of elementary quarks and leptons which have a good reason to predict two such complete vector-like families with masses in the TeV range.*

To conclude, the simplicity with which the system of three chiral and two vector-like families lead to the right gross pattern of the inter-family mass-hierarchy and the fact that the presence of the two vector-like families is fully compatible with the measurements of the light neutrinos N_ν and of the oblique electroweak parameters incline us to believe that the two vector-like

families may well exist in TeV region. The observed inter-family mass-hierarchy appears to be a strong hint in this direction. Establishing their absence in the TeV-region will clearly exclude a class of preonic theories which are based on SUSY-QCD type binding force. On the other hand, their discovery, assuming especially that their decay-pattern conforms with the one spelt out here, will first of all provide us assurance on the see-saw origin of the masses of the observed quarks and leptons –i.e., on the validity of the mass-matrix characterized by eq. (3). At the same time, considering that no other model seems to have a compelling reason to predict two vector-like families with with masses in the TeV-range, their discovery will clearly provide a strong support to the preonic approach. This in turn will provide the much needed hint as to how the superstring theories make contact with the low-energy world.

Thus we do hope that not only the LHC will be approved and built in the near future but that efforts will continue and succeed to build a future version of the SSC with $E_{cm} \geq 40 \text{ TeV}$ and the NLC e^+e^- machine with $E_{cm} \approx 1 \text{ TeV}$. Without these, some very precious discoveries, including the ones mentioned above, which are expected to lie around the corner, will never materialize.

Acknowledgments

The research of K.S.B is supported by a grant from the Department of Energy, that of JCP is supported in part by a grant from the National Science Foundation. We wish to thank A. Jawahery, J. Kim, T. Rizzo and G.A. Snow for many helpful discussions. K.S.B and H.S are thankful to the Elementary Particle Theory Group of the University of Maryland for the warm

hospitality extended to them on several occasions during the course of this work.

References

1. J.C. Pati, Phys. Lett. **B228**, 228 (1989).
2. K.S. Babu, J.C. Pati and H. Stremnitzer, Phys. Lett. **B256**, 206 (1991).
3. K.S. Babu, J.C. Pati and H. Stremnitzer, Phys. Rev. Lett. **67**, 1688 (1991).
4. M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990).
5. G. Altarelli, R. Barbieri and S. Jadach, Nucl. Phys. **B 369**, 3 (1992).
6. K.S. Babu, J.C. Pati and X. Zhang, Phys. Rev. **D 46**, 2190 (1992).
7. J.C. Pati, M. Cvetic and H. Sharatchandra, Phys. Rev. Lett. **58**, 851 (1987).
8. E. Witten, Nucl. Phys. **B 185**, 513 (1981).
9. Considering that in this paper we are interested primarily in the masses and the mixings of the vector-like families, we include here only QCD and electroweak renormalization effects to evaluate the running fermion masses, but neglect, for the sake of simplicity, radiative effects due to off-diagonal and diagonal Yukawa couplings, which follow the pattern of the fermion masses (see eqs. (2) and (3)). The Yukawa radiative effects can, of course, be important as regards predictions of the masses of the lighter chiral families (eg. m_t, m_b, m_τ etc.), especially if the

Yukawa couplings are large (order unity). But they would not affect significantly the masses and mixings of the vector-like families *relative* to those of the chiral families, because such relative effects can largely be simulated by choosing κ_λ appropriately within the range being considered (see discussions later).

10. The LEP Electroweak Working Group, Preprint CERN-PPE/93-157 (August 93).
11. K.S. Babu and J.C. Pati, Phys. Rev. **D 48**, R1921 (1993).
12. K.S. Babu, J.C. Pati and H. Stremnitzer, Phys. Lett. **B264**, 347 (1991).
13. The approximate upper limit of 150 GeV on $m_t(m_t)$, quoted in Ref. 3, corresponds to an upper limit of about 157 GeV for $m_t(\text{phys}) = m_t(m_t)[1 + 4\alpha_s/3\pi]$. The question of whether the inclusion of Yukawa radiative effects, which have been ignored in Ref. 3 [see remarks in Ref. 9], can naturally allow the upper limit on $m_t(\text{phys})$ to increase by 10 to 20% is being studied in collaboration with J. Kim. This work will be reported in a forthcoming paper.
14. E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984)
15. Z. Berezhiani, Phys. Lett. **129B**, 99 (1983); D. Chang and R.N. Mohapatra, Phys. Rev. Lett. **58**, 1600 (1987); A. Davidson and K.C. Wali, Phys. Rev. Lett. **59**, 393 (1987); S. Rajpoot, Phys. Lett. **B191**, 122 (1987).

16. B.S. Balakrishna, Phys. Rev. Lett. **60**, 1602 (1988); B.S. Balakrishna, A. Kagan and R.N. Mohapatra, Phys. Lett. **205B**, 345 (1988).
17. A.E. Nelson, Phys. Lett. **136B**, 387 (1984); S.M. Barr, Phys. Rev. Lett. **53**, 329 (1984) and Phys. Rev. **D30**, 1805 (1984); K.S. Babu and R.N. Mohapatra, Phys. Rev. **D41**, 1286 (1990).
18. P. Frampton and T. Kephart, Phys. Rev. Lett. **66**, 1666 (1991); E. Carlson and D. Land, HUTP-92/A008.
19. Having more than two vector-like families will, in general, spoil one of the most desirable features of the fermion mass-matrix of the five-family system which guarantees one zero eigenvalue [3] that corresponds to the mass of the electron family. Furthermore, subject to left-right symmetry, if there are more than two vector-like families, there would have to be four of them, i.e., two $Q_{L,R}$ and two $Q'_{L,R}$. together with their SUSY partners, they would make QCD coupling grow rapidly above 1 TeV to become confining below 10^{11} GeV. Thus, there appears to be a good reason why there should be precisely two vector-like families, corresponding to $Q_{L,R}$ and $Q'_{L,R}$, no more no less.